Monetary Policy and Debt Fragility*

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Abstract

The valuation of government debt is subject to strategic uncertainty. Pessimistic lenders, fearing default, bid down the price of debt. This leaves a government with a higher debt burden, increasing the likelihood of default and thus confirming the pessimism of lenders. This paper studies the interaction of monetary policy and debt fragility. It asks: do monetary interventions mitigate debt fragility? The answer depends in part on the nature of monetary policy, particularly the ability to commit to future state contingent actions. With commitment to a state contingent policy, the monetary authority can indeed overcome strategic uncertainty. Under discretion, debt fragility remains.

Keywords: *monetary policy, seignorage, inflation, sovereign debt, self-fulfilling debt crisis, sunspot equilibrium.*

JEL classification: E42, E58, E63, F33.

1 Introduction

This paper studies the interaction of fiscal and monetary policy in the presence of strategic uncertainty over the value of government debt. In real economies, beliefs of investors about the likelihood of government default, and hence the value of its debt, can be self-fulfilling. Pessimistic investors, fearing government default, will only purchase government debt if there is a sufficient risk premium. The resulting increase in the cost of funds makes default more likely.[1]

These results hold for real economies, in which the intervention of a monetary authority is not considered. Does this debt fragility exist in a nominal economy? The presence of a monetary authority can provide an alternative source of return through an inflation tax and perhaps use its influence to stabilize real interest

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[1] This interaction between beliefs and default is central to Calvo (1988); and other contributions that followed, including Cole and Kehoe (2000), Roch and Uhlig (2012) and Cooper (2012).

rates. Can the monetary authority act to eliminate strategic uncertainty over the value of sovereign debt? If so, will it have an incentive to do so? The answers to these questions are relevant for assessing the relevance of these results on strategic uncertainty in debt markets and for guidance on the conduct of monetary policy.

The overlapping generations model with active fiscal and monetary interventions provides a framework for analysis. The model is structured to highlight strategic uncertainty in the pricing of government debt stemming from the default choice of a government. By construction, there is an equilibrium without default, and in general there are other equilibria with state contingent default.

The monetary authority intervenes through transfers to the fiscal entity, financed by an inflation tax. The monetary intervention has a number of influences. First, the inflation tax delivers real resources to the government, thus reducing the debt burden from taxation. Second, the realized value of inflation alters the real value of debt and consequently the debt burden left to the fiscal authority. Third, it may impact expectations of future inflation and thus the tax base for seignorage.

Given these transfers and its outstanding obligations, the fiscal authority chooses to default or not. Our analysis emphasizes the dependence of this default decision - and thus the extent of strategic uncertainty - on the conduct of monetary policy.

A key issue is the ability of the monetary authority to commit to a state contingent policy. If there is complete discretion in monetary policy or if the monetary authority commits to a strict inflation target, the strategic uncertainty in the real economy is present in the monetary economy. However, if the monetary authority can commit to a particular state contingent transfer function, given an inflation target, then its intervention can stabilize debt valuations. Specifically, this policy is designed so that if the interest rate on government debt is sufficiently high, the monetary intervention will increase default probabilities enough to eliminate all equilibria except the one in which debt is risk free. Interestingly, this desired intervention does not "bail-out" the fiscal authority. Rather, the countercyclical nature of this policy induces an accommodative fiscal stance in times of low productivity. This intervention leans against negative sentiments of investors and preserve the fundamental price of debt.

Other analysis examine possible strategies for central banks to address self-fulling debt crises. Calvo (1988) extends his real economy to include a discussion of inflation as a form of partial default. He imposes an exogenous money demand and an explicit cost of inflation function that affects net output. Calvo (1988) argues that there may exist multiple equilibria in the determination of inflation and the nominal interest rate on government debt. For this analysis, there is no interaction between fiscal and monetary debt repudiation.

Corsetti and Dedola (2013) augments Calvo's framework to study the interaction of fiscal and monetary policy. Their analysis retains some of the central features of Calvo's model, including exogenous money demand and costly *ex post* inflation. They argue that monetary interventions through the printing press will not generally resolve debt fragility. But, the central bank, through its holding of government debt, can have a stabilizing influence.

Aguiar, Amador, Farhi, and Gopinath (2013) builds a nominal economy with debt roll-over crisis, as in Cole and Kehoe (2000). They investigate the optimal degree of conservativeness of the central bank (as in Rogoff (1985)) as a tool to address inefficient debt crisis. Moderate inflation aversion contains the occurrence of self-fulfilling debt crisis and restrain the inflation bias in normal times.

The paper is structured as follows. Section 2 describes the economic environment and the fiscal problem of the government. Section 3 displays debt fragility in a benchmark real economy. Section 4 defines the relevant equilibrium concept in the nominal economy and investigates the presence of debt fragility under two monetary policy framework: delegation and discretion. Section 5 characterizes a monetary policy rule that addresses the issue of debt fragility. Section 6 concludes.

2 Economic Environment

Consider an overlapping generation economy with domestic and foreign agents. Agents live two periods. Time is discrete and infinite.

There are a couple of key components of the model. First, there is a demand for money by relatively poor young agents. These agents hold money as a store of value rather than incurring a cost to save through an intermediary. Importantly, money demand is endogenous, thus making the tax base for seignorage dependent on inflation expectations of young agents.

Second the government issues debt each period and faces a choice on how to finance the repayment of its obligations. In particular, the government can tax labor income, print money or default on its debt.

The environment is structured to highlight debt fragility: there are multiple self-fulfulling values of government debt. If lenders are optimistic, the return on debt is small and the government will be more likely to choose to repay its obligations. This repayment is consistent with the low interest rate on debt. If lenders are pessimistic about repayment, the return on debt is high and consequently default is more likely. Our ultimate interest is how the possibility of financing debt obligations through an inflation tax interacts with this form of debt fragility. In this section, we describe the choices of private agents and the fiscal environment.

2.1 Private Agents

Every period, a continuum of mass 1 of domestic agents (households) is born and lives two periods. These agents consume only when old and their preferences are linear-quadratic. This restriction is introduced to neatly capture the reaction of agents to government policy choices.

Domestic agents produce a perishable good in both young and old age. Production is linear. In youth, productivity is heterogenous. A mass ν^m of agents have low productivity $z^m = 1$. A mass $\nu^I = 1 - \nu^m$ of agents have high productivity $z^I = z > 1$. In old age, productivity A is stochastic, i.i.d., and common to all old agents.[2]

Also, domestic agents have access to two technologies to store value: money or financially intermediated claims. Access to the latter is costly: agents pay a participation cost Γ for access to intermediaries. Limited financial market participation sorts agents in two groups. For convenience, we will refer to *poor* agents, who will hold only *money* in equilibrium, and *rich* agents, who hold *intermediated claims* in equilibrium.

[2] Formally, the distribution of A has full support on the closed and compact set $[A_l, A_h]$. $F(\cdot)$ is the associated cumulative distribution function, and $f(\cdot) = F'(\cdot)$.

Intermediated claims are invested either in government bonds or in a risk-free asset, e.g. storage, that delivers a real return R.

2.1.1 Poor Households

Poor households have low labor productivity $z^m = 1$ in youth. Their savings between young and old age are made only of money holdings, whose real return is given by $\tilde{\pi}$, the inverse of the gross inflation rate.[3] Their labor supply decisions in young and old age solve:

$$\max_{n,n'} E\big[u(c') - g(n')\big] - g(n) \tag{1}$$

subject to young and old age real budget constraints:

$$m = n \tag{2}$$

$$c' = A'n'(1-\tau') + m\tilde{\pi}' + t'. \tag{3}$$

In youth, poor agents supply labor n and store the proceeds with money. m are real money holdings carried on from young to old age. Return on money is given by the gross inverse inflation rate $\tilde{\pi}'$. In old age, poor agents supply labor n', which is augmented by aggregate productivity A'. τ' is the tax rate on labor income of old agents and t' a possible lump-sum transfer. Denote by n_y^m and n_o^m the optimal labor supply decision of young and old poor agents. With $u(c) = c$ and $g(n) = \frac{n^2}{2}$, labor supply decisions are:

$$n_y^m = E(\tilde{\pi}') \quad \text{and} \quad n_o^m = A'(1-\tau'). \tag{4}$$

Labor supply in both young and old age are driven by real returns to working. In youth, agents form expectations $\tilde{\pi}^e = E(\tilde{\pi}')$ on inflation, and supply labor accordingly: if agents expect high inflation, i.e. a low $\tilde{\pi}^e$, they will reduce labor supply and the associated demand for real money holding. Similarly, tax on old age labor income is distortionary: a high tax rate reduces return to working and hence the labor supply of old agents.

In contrast to, for example, Calvo (1988), money demand is endogenous in our model, reflecting a labor supply and an asset market participation decision. This is important since the impact of expected monetary interventions is to influence the magnitude of the *ex post* tax base created by money holdings. This interaction between the tax base and the tax rate generates an inflation Laffer curve.

2.1.2 Rich Households and Financial Intermediation

Rich households differ from poor agents by their productivity in youth, $z^I = z > 1$. This higher productivity induces them to pay the fixed cost Γ to access intermediated saving. Formally,

Assumption 1.

$$z^2 > \frac{R\Gamma}{R^2 - 1} > 1. \tag{\textbf{A.1}}$$

[3] We verify later that these agents prefer to save via money rather than costly intermediaries in equilibrium.

This parametric restriction ensures that young agents save via the financial sector for any positive expected inflation rate.[4]

The rich solve:

$$\max_{n,n'} E\big[u(c') - g(n')\big] - g(n) \tag{5}$$

subject to young and old age real budget constraints:

$$m + s = zn - \Gamma \tag{6}$$

$$s = b^I + k \tag{7}$$

$$c' = A'n'(1-\tau') + \tilde{\pi}'m + (1+i)\tilde{\pi}'b^I + Rk + t'. \tag{8}$$

In youth, rich agents supply labor n and produce zn. After incurring the fixed cost Γ, they invest a per capita amount s in intermediated claims. These claims are invested in government bonds b^I and risk-free assets k so that $s = b^I + k$, where b^I denotes the per-capita holding of government debt of domestic rich agents. When old, they supply n', contingent on the realization of A' and the tax rate τ'.

Finally, given linear utility of consumption, the portfolio decision between intermediated saving s and money holding m is only driven by expected returns. As long as expected return on money holding $\tilde{\pi}^e$ is strictly inferior to the real return R on the risk-free asset, rich households do not hold money. The portfolio for intermediated savings will include both government debt and risk-free asset as long as the expected return on government debt equals that on the asset:

$$(1+i)\tilde{\pi}^e\big(1 - P^d\big) = R \tag{9}$$

where P^d is the probability of default, determined in equilibrium. We refer to this as the 'no arbitrage condition'.

Denote by n_y^I and n_o^I the optimal labor supply decisions of intermediated agents in young and old age. The solution to (5) implies:

$$n_y^I = Rz \;\text{ and } n_o^I = A'(1-\tau'). \tag{10}$$

Labor supply n_y^I of young agents is determined by the expected return R on intermediated savings. In old age though, the effective return on intermediated savings will depend on the realized inverse inflation rate $\tilde{\pi}'$, the nominal interest rate i and the default decision of the government.[5]

2.1.3 Foreign Households

In addition to domestic agents, there are also foreign households who hold domestic debt. They are like rich households in that they save through intermediaries that hold government debt. The details of the foreign economy are not important for this analysis except that foreign households are risk neutral and have access to domestic debt as a store of value. In equilibrium, they hold a fraction $(1-\theta)$ of domestic debt.[6]

[4] We verify this in characterizing equilibria.

[5] Especially, in case of default, the old age budget constraint of rich agent (8) becomes $c' = A'n'(1-\tau') + \tilde{\pi}'m + Rk + t'$.

[6] Given the indifference of risk neutral agents regarding their portfolio of government debt and storage, θ is not determined in equilibrium. Thus equilibria will be characterized for given values of θ.

2.2 The Government

The government is composed of a fiscal treasury and a central bank. Every period, it has to finance a constant and exogenous flow of real expenses g. Government expenditures do not enter into agents utility. To finance these expenses, it can raise taxes on old agent labor income, print money and issue nominal debt B'.[7] Alternatively, it can default on its inherited debt obligation. We describe these choices.

Under repayment, the real budget constraint of the government is:

$$(1+i)\tilde{\pi}b + g = \tau\big(\nu^m A n_o^m(\tau) + \nu^I A n_o^I(\tau)\big) + \frac{\Delta M}{P} + b'. \tag{11}$$

The left hand side contains the real liabilities of the government, net of realized inflation $\tilde{\pi}$. On the right hand side, $n_o^j(\tau)$ is the labor supply decision of old agents of type $j \in \{I, m\}$, $M = \nu^m P m$ is total money supply, P is the price level and $\nu^m m$ is the inherited aggregate real money base.

The focus of the paper is on the occurrence of self-fulfilling debt crisis in nominal economies under different monetary policies. Hence, we assume $g = b'$, so that new expenses are financed exclusively by debt. With this restriction, fiscal policy has no intergenerational element. Instead, debt created when agents are young is paid for or defaulted on when these agents are old.[8] We return to this restriction in our concluding comments. So the government budget constraint under repayment becomes generation specific:

$$(1+i)\tilde{\pi}b = \tau\big(\nu^m A n_o^m(\tau) + \nu^I A n_o^I(\tau)\big) + \frac{\Delta M}{P}. \tag{12}$$

If the government defaults, then it avoids debt obligations of the current old. In keeping with the generational view of the budget constraint, any money creation in the current period is transferred lump-sum to the current old. The amount of this transfer will depend on the monetary regime.

With $\tilde{\pi}^e$ denoting the expected inverse inflation rate, formed previous period by private agents, and σ the money creation rate, the real resources collected from seignorage are:

$$\frac{\Delta M}{P} = \nu^m m \frac{\sigma}{1+\sigma} = \nu^m \tilde{\pi}^e \frac{\sigma}{1+\sigma}. \tag{13}$$

The second equality comes from the labor supply decision of a young agent (4) which links real money demand and inflation expectation. The money creation rate σ and inverse inflation rate $\tilde{\pi}$ are related by $\tilde{\pi} = \frac{1}{1+\sigma}$.[9]

Using labor supply decisions (4) and (10) and the expression (13), the government budget constraint (12) becomes:

$$(1+i)\tilde{\pi}b = A^2(1-\tau)\tau + \nu^m \tilde{\pi}^e (1-\tilde{\pi}). \tag{14}$$

Under repayment, real debt liability, net of inflation, is serviced with labor taxes on old agents and an inflation tax on old money holders, given the real money tax base $\nu^m \tilde{\pi}^e$.

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[7] The assumption of no taxation of income when young implies the government policy has no intergenerational dimension. This is just a simplification that allows to neatly disentangle demand for money, for intermediated claims and labor supply driven by taxation.

[8] This use of generational budget balance appears in Chari and Kehoe (1990) and Cooper, Kempf, and Peled (2010), for example. An alternative, as in Cole and Kehoe (2000), could add more strategic uncertainty through debt rollover.

[9] This is an equilibrium property that we verify below.

Note that both realized and expected inflation are present in (14). The realized inflation determines both the real debt burden and the tax extracted from the money holders. High inflation benefits all agents by reducing the real value of the debt. But it is the money holders who pay the inflation tax. Though current inflation has no effect on their labor supply, the expected inflation influences the money demand of young agents in the previous period and thus the tax base in the present period.

To establish a useful benchmark, under perfect foresight, the level of revenues obtained from seignorage are given by $\nu^m \tilde{\pi}(1-\tilde{\pi})$. This is maximized at $\tilde{\pi}^L \equiv \frac{1}{2}$ which is the top of the "Laffer curve". At $\tilde{\pi} > \tilde{\pi}^L$, a reduction in $\tilde{\pi}$ (i.e. an increase in the rate of inflation) will increase revenues.

2.2.1 Strategic Default

The government has the possibility to fully renege on its debt, but default is costly. There are two costs of default for domestic agents. First, direct costs of default are born by old rich agents, who hold a fraction θ of government debt. Second, if the government repudiates its debt, the country suffers from a deadweight loss, as usually assumed in the literature on strategic default.[10] Formally, aggregate productivity contemporaneously drops by a proportional factor γ.

As the government budget constraint holds over time for a given generation, a decision to default on period t debt has no direct effect on future generations. Further, the analysis abstracts from any reputation costs of default. Hence, default affects only the welfare of current old agents, who otherwise are taxed via seignorage or labor tax.

The government weights the welfare burden of tax distortions against the direct costs and penalty induced by the default decision. Denote by $W^r(\cdot)$ the welfare of the economy under *repayment* and by $W^d(\cdot)$ under *default*. The decision to default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

Given aggregate productivity A, nominal interest rate i, real money tax base m, tax rate τ and inverse inflation rate $\tilde{\pi}$ that satisfy (12), the welfare criterion $W^D(\cdot)$ for $D \in \{r, d\}$ is:

$$W^D(A, i, m, \tau, \tilde{\pi}) = \nu^m \Big(c_o^m(D) - \frac{n_o^m(D)^2}{2} \Big) + \nu^I \Big(c_o^I(D) - \frac{n_o^I(D)^2}{2} \Big). \tag{15}$$

Specifically, under repayment, $D = r$, the welfare of old agents is:[11]

$$W^r(A, i, m, \tau, \tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^m m \tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^I R(Rz^2 - \Gamma). \tag{16}$$

The option to default, $D = d$, triggers penalties but no tax need be raised. In this case, the welfare of old agents becomes:

$$W^d(A, i, m, \tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m m \tilde{\pi}^d - R\theta b + \nu^I R(Rz^2 - \Gamma) + T \tag{17}$$

where $T = \nu^m m(1 - \tilde{\pi}^d)$ is the aggregate lump sum transfer to old agents that implement the inflation choice $\tilde{\pi}^d$.

[10] Penalties and direct sanctions are central theoretic concepts for enforcement of international asset trade. See the seminal work by Eaton and Gersovitz (1981). Also, see Bulow and Rogoff (1989). For an extensive review, see Eaton and Fernandez (1995).

[11] Computations are detailed in Appendix 7.1.

2.3 Assumptions

The following two assumptions are used for characterizing equilibria. The first places a lower bound on γ to ensure that default is costly, especially when no debt is held by domestic agents.

Assumption 2.

$$\frac{A_l^2\gamma(2-\gamma)}{2} > \nu^m. \tag{A.2}$$

The second assumption ensures that the fundamentals of the economy are compatible with a risk-free outcome, i.e. given the real level of debt b, a real interest rate of R, all debt held abroad and no revenues from seignorage, the debt will be repaid for all A. That is, there is a solution to (9) without default. Formally,

Assumption 3. $b < \bar{b}$ *where*

$$\bar{b} = \frac{A_l^2(1-\gamma)\gamma}{R}. \tag{A.3}$$

Assumption 3 is stated for $\tilde{\pi}^* = 1$, where there is no seignorage, and all debt is held by foreigners.[12] The presence of an equilibrium without default provides a convenient benchmark for the analysis.

3 Debt Fragility in a Real Economy

To explicitly illustrate debt fragility, first consider this economy without money and nominal quantities. The equilibrium in the monetary economy will be constructed on the foundation of the multiplicity in the real economy.

The government issues real debt and can raise only taxes on labor income. Its budget constraint under repayment is simply:

$$(1+i)b = A^2(1-\tau)\tau. \tag{18}$$

Let τ be the smallest tax rate satisfying (18) so that tax revenues are locally increasing in the tax rate.

Private agents can save only through intermediation. For simplicity, set $\Gamma = 0$ so that there are no costs associated to saving through the holding of government debt. Further, assume $\theta = 0$ so that all debt is held by foreigners and all domestic saving is through storage.

The labor supply choices of the rich are given by (10). Since the poor access the intermediary for saving, their labor supply decisions are given by

$$n_y^m = R \quad \text{and} \quad n_o^m = A'(1-\tau'). \tag{19}$$

The government defaults whenever $\tilde{W}^d(\cdot) \geq \tilde{W}^r(\cdot)$, where these values for the real economy are defined by:

$$\tilde{W}^r(A,i,\tau) = \frac{\left[A(1-\tau)\right]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2. \tag{20}$$

[12] This assumption is derived using the government budget constraint with no fiscal resources from seignorage and all debt is held by foreigners ($\theta = 0$). It implies that there will be an equilibrium without default risk when some of the debt is held by domestic agents and when money printing does provide resources to the fiscal authority. Indeed, domestic holding of public debt or a higher money printing rate relaxes the willingness of the fiscal authority to default rather than repay its debt.

and

$$\tilde{W}^d(A,i) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2 \tag{21}$$

For the real economy with $\theta = 0$, the government will default whenever $\tau > \gamma$. Equivalently, using (18), the government defaults for any realization of $A < \bar{A}$, where $\bar{A}$ satisfies:

$$\bar{A}^2 = \frac{(1+i)b}{\gamma(1-\gamma)}. \tag{22}$$

This expression defines $\bar{A}(i)$, the default threshold of the government as a function of the real interest rate i.

The probability of default given i is $F\big(\bar{A}(i)\big)$. Using this, the no-arbitrage condition (9) becomes:

$$(1+i)\Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{23}$$

This equation may have several solutions.[13] Default arises both because of fundamental shocks (low A) and strategic uncertainty: the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability. Hence the multiplicity.

Proposition 1. *If government debt has value, then there are multiple interest rates consistent with (23).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold, $\bar{A}$ solving (22) and (23). Inserting (22) in (23) yields

$$\bar{A}^2\big(1 - F(\bar{A})\big) = \frac{Rb}{\gamma(1-\gamma)}. \tag{24}$$

Any $\bar{A}$ solving this equation is an equilibrium.

Denote by $G(A)$ the left side and by Z the right side of (24). $G(\cdot)$ is continuous on $[A_l, A_h]$, $G(A_l) = A_l^2 > 0$ and $G(A_h) = 0$.

Consistent with Assumption 3, if $Z < A_l^2 = G(A_l)$, there is a risk free interest rate: $\bar{A} = A_l$ and $(1+i) = R$ is a solution to (24). Also, by continuity of $G(\cdot)$, there is $\bar{A} \in (A_l, A_h)$ such that $G(\bar{A}) = Z$. Hence there is also an interest rate that carries a risk-premium and that solves the no-arbitrage condition.

Relaxing Assumption 3, if $Z > A_l^2 = G(A_l)$, all equilibria include default risk and thus a risk premium. If b is very high, then Z will be large as well and there may be no equilibrium in which debt is valued. If debt is valued so that there is a solution to (24), then $G(A) > Z$ for some $A > A_l$. Again, by continuity, there is a second equilibrium. ∎

The underlying source of strategic uncertainty for the real economy is aptly captured by Proposition 1 for the real economy. It is a building block for the analysis of a monetary economy. The subsequent analysis allows debt to be held both internally and externally.

4 Debt Fragility in a Monetary Economy

This section studies the interaction of monetary interventions and debt fragility. Intuitively, monetary interventions can act via two channels. First, it can collect seignorage taxes and supplement the resources

[13] This source of multiplicity is at the heart of Calvo (1988) and Cooper (2012).

collected through labor taxation. Second, by adjusting the realized inflation rate, it can lower the real value of debt.

But, there are potential resource costs of funding the government through the inflation tax: young agents perceiving high inflation in the future will work less, reducing the monetary tax base. This effect though depends on the extent of discretion in the conduct of monetary policy. Also, the mean inflation rate $\tilde{\pi}^e$ is priced into the nominal interest rate i, which makes attempt to deliver surprise inflation difficult.

Accordingly, this section of the paper is constructed around two polar cases, distinguished by the ability of the monetary authority to commit.

i. *Monetary delegation*: monetary policy decisions are made by an independent central bank that pursues a known and explicit rule, independent of fiscal considerations. We consider the case of strict inflation targeting: the central bank announces one period in advance an inflation rate and delivers it.

ii. *Monetary discretion*: monetary and fiscal decisions are linked. Given the state of the economy, money creation and taxes are set so as to minimize welfare costs and tax distortions of old agents given a budget constraint. Default is also chosen optimally *ex post*.

Before analyzing debt fragility under these monetary regime, we formally define the equilibrium concept of the nominal economy.

4.1 State Variables and Equilibrium Definition

The strategic uncertainty identified in Proposition 1 is modeled through a sunspot variable, denoted s, that corresponds to confidence of domestic and foreign households about the repayment of government debt next period.

- If $s = s^o$, agents are "optimists" : they coordinate on the risk free (fundamental) price of the government debt.

- If $s = s^p$, agents are "pessimists" : they coordinate on higher risk / lower price equilibria with partial default.

In the event there is a unique equilibrium price, then the fundamental price obtains regardless of the sunspot realization. Note that we only consider cases where debt has value.[14]

The state of the economy is $\mathcal{S} = (A, i, m, s, s_{-1})$. Aggregate productivity, A, is realized and directly affects the productivity of the old. There are two endogenous predetermined state variables, m and i, respectively real money holdings of current old agents and nominal interest rate on outstanding public debt. Both the sunspot shock last period, s_{-1}, and the current realization, s, may impact fiscal policy, monetary policy and the choices of private agents.

Definition 1. *A stationary rational expectations equilibrium is given by:*

[14]The case of "market shutdown", where debt has no value, is not of direct interest for our analysis.

- *Given state contingent monetary and fiscal policies* $(\{\tau(S), \tilde{\pi}(S), D(S)\})$*, private agents form rational expectations in youth, supply labor in young and old age to solve (1) and (5) subject to their respective budget constraints (2,3) and (6,8).*

- *The government maximizes its welfare criterion and generates the optimal policy* $(\{\tau(S), \tilde{\pi}(S), D(S)\})$.

- *All markets clear (goods, money, bonds) in all states.*

- *In equilibrium, expectations on inflation* $\tilde{\pi}^e$ *are consistent with the policy delivered* $\{\tilde{\pi}(\mathcal{S})\}$*:* $E(\tilde{\pi}(\mathcal{S})) = \tilde{\pi}^e$.

The welfare criterion of the government will depend on the monetary policy framework, as detailed below. Also, we characterize equilibria for given θ, share of government debt held by domestic agents, as its value is not pinned down in equilibrium. The polar cases of delegation and discretion are studied within this framework; the conduct of monetary policy determines what the government takes as given in choosing its policy.[15]

4.2 Monetary Delegation

In this institutional structure, the treasury has discretionary power over fiscal policy, choosing fiscal policy *ex post* given the monetary intervention. In contrast, the monetary authority is endowed with a commitment technology. We find that under strict monetary delegation to an inflation target, debt fragility remains.[16]

One interpretation of this structure is that the government of an individual country delegates its monetary policy to an independent central bank, by joining a monetary union for instance. The central bank of the union pursues an independent policy of strict inflation targeting and the fiscal authority is left with discretionary tax policy choices (taxes or default).

Specifically, the central bank announces one period in advance its inflation target $0 < \tilde{\pi}^* \leq 1$ and delivers it by printing money.[17] By doing so, the central bank does not accommodate country-specific technological shocks. Revenues from seignorage are transferred to the treasury. Formally, the policy of the central bank is:

$$\tilde{\pi}(\mathcal{S}) = \tilde{\pi}^* \qquad \forall \mathcal{S}. \tag{25}$$

As the central bank is bound to deliver its inflation target $\tilde{\pi}^*$, agents' expectations are $\tilde{\pi}^e = \tilde{\pi}^*$.[18]

[15] Aguiar, Amador, Farhi, and Gopinath (2013) and Corsetti and Dedola (2013) study discretionary monetary authorities. Our analysis will highlight particular forms of commitment by the central bank.

[16] The analysis with commitment is extended beyond strict inflation targeting in Section 5.

[17] The determination of the optimal inflation target $\tilde{\pi}^*$ is not part of the present analysis. The model could provide a positive theory of inflation, where the inflation target would be set to minimize distortions associated to tax revenues. Given the Laffer curve property of seignorage revenue, any inflation target $0 < \tilde{\pi}^* < \tilde{\pi}^L$ is inefficient, but this does not affect the essential results regarding debt fragility.

[18] In particular, if there is default, the monetary authority prints money and transfer it to old agents to meet this target.

4.2.1 Multiple Solutions to the No-arbitrage Condition

The default decision in the monetary economy has the same monotonicity property as in the real economy: if the government defaults for a given realization of technology $\bar{A}$, then it would default for any lower realization $A \leq \bar{A}$.

Lemma 1. *Under Assumption 2, given a level of real expenses* $(1+i)\tilde{\pi}^* b$*, there is a unique* $\bar{A}(i) \in [A_l, A_h]$ *such that if* $A \leq \bar{A}(i)$*, then the treasury defaults on its debt. Otherwise it repays its debt.*

Proof. Given a nominal interest rate i, the decision to repay or default on debt is given by $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$, where the relevant welfare criteria are given by (16) and (17). Hence, a point of indifference between default and repayment, $\bar{A}(i)$ solves:

$$\frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} = (1+i)\tilde{\pi}^*\theta b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*) \tag{26}$$

where τ satisfies the government budget constraint:

$$(1+i)\tilde{\pi}^* b = A^2(1-\tau)\tau + \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*). \tag{27}$$

Denote by $G(A)$ the left side of (26). Clearly if $G(A)$ is monotonically decreasing in A, then the default decision satisfies the desired cut-off rule. Rewrite $G(A)$ as follow:

$$G(A) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \frac{A^2\tau(\tau-2)}{2}.$$

Using the government budget constraint:

$$\begin{aligned} G(A) &= \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \left[(1+i)\tilde{\pi}^* b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*)\right]\frac{(\tau-2)}{2(1-\tau)} \\ &= \frac{A^2\gamma(\gamma-2)}{2} - \left[(1+i)\tilde{\pi}^* b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*)\right]\frac{(\tau-2)}{2(1-\tau)}. \end{aligned}$$

The first term is negative since $\gamma < 1$. If seignorage revenues are enough to service debt, then no tax need be raised and $\bar{A}(i) = A_l$ by Assumption 2. Otherwise, $(1+i)\tilde{\pi}^* b - \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*) > 0$. Finally, we need to derive the monotonicity of $\frac{\tau-2}{1-\tau}$ with respect to A. Its derivative is:

$$\frac{-1}{(1-\tau)^2}\frac{d\tau}{dA} > 0 \tag{28}$$

which is positive since $\frac{d\tau}{dA} < 0$ for the lowest value of τ that solves the budget constraint. Overall, we have $G'(A) < 0$. Hence, the cut-off value $\bar{A}(i)$ is unique and default occurs if and only if $A \leq \bar{A}(i)$.

Note that if $\bar{A}(i) \leq A_l$, then debt is risk free. Finally, $\bar{A}(i) = A_h$ is inconsistent with the assumption that debt has value. ∎

From this result, the probability of default P^d becomes $F(\bar{A}(i))$. This probability also implicitly depends on $\tilde{\pi}^*$, which appears on the right side of (26). Altogether, an interest rate for the government debt solves

$$(1+i)\tilde{\pi}^*\Big(1 - F\big(\bar{A}(i)\big)\Big) = R. \tag{29}$$

This expression outlines the interplay between beliefs, probability of default and best-response of the government. As in the real economy, the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability.

Lemma 2. *Under Assumptions 2 and 3, for any inflation target $0 < \tilde{\pi}^* \leq 1$, there are multiple interest rates that solve the no-arbitrage condition (29).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold $\bar{A}$. Importantly, an equilibrium is such that *beliefs* of investors are consistent with the *best response* of the government.

If investors believe that the government defaults with probability $P^d = F(\bar{A})$, they charge an interest rate i. Hence, for all i, the beliefs induce $\bar{A}^b(i)$, the default threshold consistent with P^d:

$$(1+i)\tilde{\pi}^*\big(1 - F(\bar{A})\big) = R \Rightarrow \ \bar{A}^b(i). \tag{30}$$

Given i, the government decision to repay or default induces $\bar{A}^g(i)$, the realization of A for which the government is indifferent between default and repayment:[19]

$$\Delta(A,i) = W^d(A,i) - W^r(A,i) = 0 \Rightarrow \bar{A}^g(i). \tag{31}$$

An equilibrium is such that $\bar{A}^b(i) = \bar{A}^g(i)$. The nominal interest rate i can takes value on $[\underline{i}, +\infty)$ where $\underline{i}$ is the nominal interest rate consistent with risk-free debt. Formally, it satisfies $(1+\underline{i})\tilde{\pi}^* = R$. We study the monotonicity properties of $\bar{A}^b(\cdot)$ and $\bar{A}^g(\cdot)$.

The default threshold $\bar{A}^b(i)$ induced by belief of investors has the following properties. First, $\bar{A}^b(\underline{i}) = A_l$: if investors charge $\underline{i}$, it means that they expect no default. Second, differentiating (30) with respect to $\bar{A}$ and i, one gets:

$$\frac{d\bar{A}^b(i)}{di} = \frac{\big(1 - F(\bar{A})\big)}{f(\bar{A})(1+i)} > 0, \tag{32}$$

since $F'(\cdot) > 0$. Finally, $\lim\limits_{i \to +\infty} \bar{A}^b(i) = A_h$.

The best response of the government to i is captured by $\bar{A}^g(i)$, the default threshold. Given Assumption 3, for low values of i, debt is risk free.[20] Hence, there is $\epsilon > 0$ such that $\bar{A}^g(\underline{i} + \epsilon) = A_l$. Second, by differentiating (26) with respect to $\bar{A}$ and i, one gets:

$$\tilde{\pi}^* b\Big[\frac{1-\tau}{1-2\tau} - \theta\Big]di + \bar{A}\Big[(1-\gamma)^2 - \frac{(1-\tau)^2}{1-2\tau}\Big]d\bar{A} = 0. \tag{33}$$

The factor of di is positive since $\frac{1-\tau}{1-2\tau} > 1$ and the factor of $d\bar{A}$ is negative since $\frac{(1-\tau)^2}{1-2\tau} > 1$. Hence:

$$\text{if } \bar{A}^g(i) \in (A_l, A_h), \text{ then } \frac{d\bar{A}^g(i)}{di} > 0.$$

Finally, there is an upper bound $\bar{i}$ such that default occurs for all A if $i \geq \bar{i}$:

$$\forall i > \bar{i}, \ \bar{A}^g(i) = A_h.$$

By continuity of the the function $\bar{A}^g(\cdot)$ and $\bar{A}^b(\cdot)$, there is a value $i > \underline{i}$ that satisfies $\bar{A}^g(i) = \bar{A}^b(i)$. ∎

[19] Lemma 1 established that this threshold is unique. To determine $\Delta(A,i)$ from (16) and (17) ,set $\tilde{\pi} = m = \tilde{\pi}^*$ and set τ from (27) if the government chooses to repay.

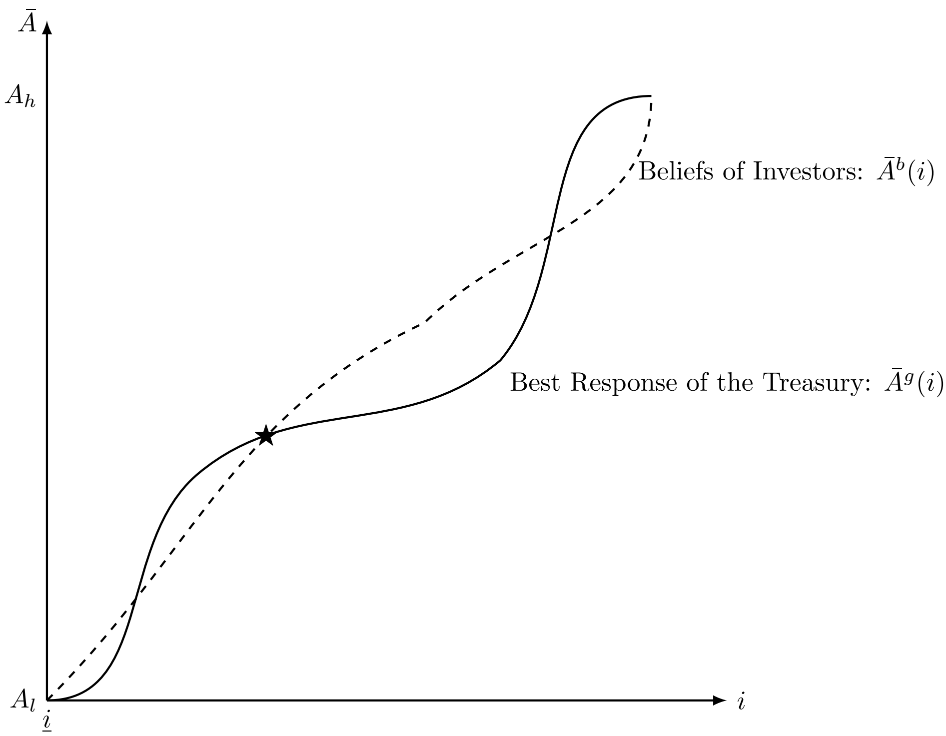
[20] Relaxing Assumption 3 and allowing a fundamental equilibrium with positive probability of default does not change the central result that several interests rate are compatible with the no-arbitrage condition. This is explicit in the real environment, as in Proposition 1, and can be established in the nominal economy as well.

The monotonicity properties of $\bar{A}^g(i)$ and $\bar{A}^b(i)$ are summarized in Figure 2. Overall, the proof shows that in addition to the no-default equilibrium, there is $\bar{A} \in (A_l, A_h)$ and $i > \underline{i}$ that form an equilibrium with a positive probability of default.

Under Assumption 3, there is always an equilibrium with certain repayment. In addition, there will exist an equilibrium in which the debt is never repaid and, accordingly, investors place zero probability on repayment.[21] Lemma 2 characterizes additional interior equilibria in which default arises with a probability less than one.

Figure 2 illustrates the multiplicity of equilibria, including three interior equilibria. The equilibrium labeled ★ is a locally stable equilibrium with a positive probability of default. Here local stability refers to best-response dynamics. Local stability is used for comparative statics.

Figure 1: Multiplicity of Interest Rates under Monetary Delegation



This figure represents the mapping from interest rate i to default threshold $\bar{A}$, both for investors and the fiscal authority. Investors associate an interest rate i to a default threshold via the probability of default in the no-arbitrage condition. This is the dashed line. Given the interest rate i, the optimal decision of the fiscal authority to service its debt or default is captured by the default threshold, indicated by the solid line. An equilibrium is reached when beliefs of investors are consistent with the best-response of the fiscal authority. The figure highlights the existence of several equilibria, one of them being risk-free. The equilibrium indicated with a ★ is locally stable under best response dynamics.

This lemma provides the basis for a sunspot equilibrium in which the value of government debt is de-

[21] As mentioned previously, we discard this “market shutdown” case, which always exists.

pendent upon the beliefs of investors. In equilibrium there are sunspot dependent variations in employment, output and consumption.

Proposition 2. *Under Assumption 2 and 3, for any* $0 < \tilde{\pi}^* \leq 1$*, there is a sunspot equilibrium with the following characteristics:*

- *If* $s_{-1} = s^o$*, the interest rate to the government security is risk free and the treasury reimburses with probability 1.*
- *If* $s_{-1} = s^p$*, the interest rate incorporates a risk-premium and the treasury defaults on its debt with positive probability.*

Proof. The characterization of the sunspot equilibrium directly comes from Lemma 2 and the existence of several interest rates compatible with the no-arbitrage condition in equilibrium. We describe the optimal behavior of agents consistent with the equilibrium definition.

As $\tilde{\pi}^e = \tilde{\pi}^* \in (0, 1]$, poor agents save only with money holding and rich young agents invest in intermediated claims. Indeed, consider a young household with productivity mark-up z. It can either save with money holding or via the financial sector, incurring the fixed cost Γ.

If it chooses to hold money, its labor supply when young is $n = z\tilde{\pi}^e$, its real demand for money holding is $zn = z^2\tilde{\pi}^e$ and the net expected contribution to consumption: $\left(z\tilde{\pi}^e\right)^2$. If it chooses the intermediated savings, its labor supply when young is $n = Rz$, its savings net of the intermediation cost $s = Rz^2 - \Gamma$ and the net expected contribution to consumption: $R(Rz^2 - \Gamma)$. Hence, intermediated saving dominates money holding if and only if:

$$z^2 > \frac{R\Gamma}{R^2 - (\tilde{\pi}^e)^2}$$

which is true for any $\tilde{\pi}^e \in (0, 1]$ as long as Assumption 1 holds. An aggregate fraction $\theta \in [0, 1]$ of the government security is being held by domestic rich agents.

If $s_{-1} = s^o$, then young agents form expectations $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They supply labor accordingly. Consequently, the interest rate on debt satisfies the no-arbitrage condition (9) with $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i, seignorage revenue $\nu^m\tilde{\pi}^*(1-\tilde{\pi}^*)$ and using Assumption 3, the optimal policy of the treasury is then to raise labor taxes τ for all A so as to satisfy its budget constraint and repay its debt. All markets clear.

For the case $s_{-1} = s^p$, we outline only differences with the previous case. From Lemma 2, there is an interest rate i that carries a risk premium and satisfy the no-arbitrage condition, such that $(1 + i)\tilde{\pi}^* > R$. Young agents form expectations $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They price the government debt according to $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i and seignorage revenue $\nu^m\tilde{\pi}^*(1 - \tilde{\pi}^*)$, there is a unique threshold $\bar{A}(i)$ such that the optimal policy of the treasury is to raise labor taxes τ for all $A \geq \bar{A}(i)$ to satisfy its budget constraint and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F\big(\bar{A}(i)\big)$. ∎

Aguiar, Amador, Farhi, and Gopinath (2013) find a similar result if there is a very high perceived cost of inflation: when the central bank is very inflation averse, it never chooses to inflate the nominal value of debt, it is committed to no inflation, converting nominal debt into real debt. Roll-over crisis occur for higher range of debt ("crisis zone").

Overall this section, particularly Proposition 2, makes clear that the debt fragility in real economics, summarized in Proposition 1, extends to economies with nominal debt. In effect, the inflation target of the monetary authority converts the nominal obligation to a real one thus reducing the real debt burden left to the fiscal authority without eliminating the underlying strategic uncertainty. The choice of the inflation target does not allow the monetary authority to peg the real interest rate. Instead the real interest rate on debt continues to reflect the sentiments of investors.

This does not imply though that the equilibrium is independent of the inflation target. An increase in the inflation target, i.e. a reduction in $\tilde{\pi}^*$, will provide seignorage revenues and thus relax the fiscal burden. This implies a lower probability of default and thus a lower risk premium.

Proposition 3. *In the equilibrium characterized in Proposition 2, for $\tilde{\pi}^* \geq \tilde{\pi}^L$, an increase in the target inflation rate will increase seignorage and lower the probability of default if and only if the equilibrium is locally stable.*

Proof. The proof relies on three expressions. The point of indifference between repayment and default, (i.e. the default threshold $\bar{A}$) is given in (26), the government budget constraint given in (27), and the no arbitrage condition, given in (29). These are all evaluated at a given inflation target and thus $\tilde{\pi}^*$. Substituting the no-arbitrage condition into (26) gives:

$$\frac{[\bar{A}(1-\gamma)]^2}{2} - \frac{[\bar{A}(1-\tau)]^2}{2} + \nu^m \tilde{\pi}^*(1-\tilde{\pi}^*) - \frac{R\theta b}{1-F(\bar{A})} = 0, \tag{34}$$

where τ satisfies the government budget constraint evaluated at $\bar{A}$.

Taking the derivative of (34) w.r.t. $\bar{A}$ and π^*:

$$\Big[\bar{A}(1-\gamma)^2 - \bar{A}(1-\tau)^2 + \bar{A}^2(1-\tau)\tau_A - \frac{R\theta b f(\bar{A})}{\big(1-F(\bar{A})\big)^2}\Big] d\bar{A} + \Big[\bar{A}^2(1-\tau)\tau_{\tilde{\pi}^*} + \nu^m(1-2\tilde{\pi}^*)\Big] d\pi^* = 0, \tag{35}$$

where τ_A and $\tau_{\tilde{\pi}^*}$ are given by the derivative of the government budget constraint evaluated in $\bar{A}$. Substituting the no-arbitrage condition into the budget constraint and taking the derivative w.r.t. τ, π^*, A, one gets:

$$\tau_A = \frac{1}{A^2(1-2\tau)}\Big[\frac{Rbf(A)}{\big(1-F(A)\big)^2} - 2A(1-\tau)\tau\Big] \qquad \tau_{\tilde{\pi}^*} = -\frac{\nu^m(1-2\tilde{\pi}^*)}{A^2(1-2\tau)}. \tag{36}$$

Rearranging (35), one gets:

$$\Big[\bar{A}(1-\gamma)^2 - \bar{A}\frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(\bar{A})}{\big(1-F(\bar{A})\big)^2}\Big(\frac{1-\tau}{1-2\tau} - \theta\Big)\Big] d\bar{A} + \Big[\nu^m(1-2\tilde{\pi}^*)\Big(1 - \frac{1-\tau}{1-2\tau}\Big)\Big] d\tilde{\pi}^* = 0 \tag{37}$$

The factor of $d\tilde{\pi}^*$ is positive since $\tilde{\pi}^* \geq \tilde{\pi}^L$ and $\frac{1-\tau}{1-2\tau} > 1$. Hence the sign of the factor of $d\bar{A}$ is critical to derive the response of the default threshold to a change in the inflation target.

This sign is determined by the condition of local stability. An equilibrium is locally stable under best response dynamics if and only if $\frac{d\bar{A}^g(i)}{di} < \frac{d\bar{A}^b(i)}{di}$. Rewriting (32) with the no-arbitrage condition and using (33), the condition for local stability becomes:

$$\bar{A}(1-\gamma)^2 - \bar{A}\frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(A)}{\big(1-F(A)\big)^2}\Big[\frac{1-\tau}{1-2\tau} - \theta\Big] < 0 \tag{38}$$

Hence under local stability, $\frac{d\bar{A}}{d\pi^*} > 0$. ∎

Proposition 3 is essentially a comparative statics result and thus holds for only a subset of equilibria, i.e. those that are locally stable under best response dynamics. A locally stable equilibrium is indicated in Figure 2 and refers to the determination of the interest rate on debt and the default cut-off. The relative slopes of the two curves at this point are used in the proof of Proposition 3.

4.3 Monetary Discretion

Does monetary discretion insulate against debt fragility? Intuitively, a discretionary policy maker could adjust inflation and seignorage to accommodate variations in the price of government debt driven by strategic uncertainty.

In a monetary discretion regime, the government has full discretionary power over both monetary and fiscal policy. It designs its policy $(\tau, \tilde{\pi}, D)$ every period, as a best response to realized productivity shock A, the sunspots (s, s_{-1}) and predetermined variables of the economy m and i. The government maximizes the welfare of home agents. This is, in effect, the same as minimizing the cost of its policy to taxpayers, hence to old agents, since they contribute to government's resources via the tax on labor income and seignorage on money holding.

In an environment with discretion, money creation provides an *ex post* source of revenue without creating any distortion. This low social cost of revenue ought to reduce the likelihood of default and stabilize debt values.

But, an essential element of this environment is the interaction between expected and realized inflation. Specifically, if agents anticipate high inflation ($\tilde{\pi}^e$ low), they would reduce labor supply in youth and their real money holdings m accordingly. To collect revenues from seignorage, the central bank then has to deliver a higher inflation rate (low $\tilde{\pi}(A, i, m)$), consistent with the initial beliefs of agents. Hence, under discretion, the capacity of the central bank to support a stressed fiscal authority may be compromised by the strategic complementarity between expected inflation and delivered inflation: if agents anticipate the willingness of the central bank to resort to inflation, the real money tax base would decrease, which in turn reduces the capacity of the central bank to intervene.

We first characterize the policy choices of a discretionary policy maker and then investigate the properties of the equilibrium under discretionary monetary policy.

4.3.1 Choice Problem of a Discretionary Government

Given the productivity shock A, real money holding m and nominal interest rate i on debt, the government chooses to default $D = d$ or repay $D = r$ its debt:

$$D \in \{r, d\} = \text{argmax}\Big[\max_{\tau, \tilde{\pi}^r} W^r(A, i, m, \tau, \tilde{\pi}^r), W^d(A, i, m, \tilde{\pi}^d)\Big] \tag{39}$$

subject to:

$$(1 + i)\tilde{\pi}^r b = A^2(1 - \tau)\tau + \nu^m m(1 - \tilde{\pi}^r) \qquad \text{(if repayment)} \tag{40}$$

$$\tau \geq 0, \qquad \tilde{\pi}^d \in [\underline{\tilde{\pi}}, 1]. \tag{41}$$

Here, $W^D(\cdot)$ is given by (16) for $D = r$ and by (17) for $D = d$.

The first constraint is the government budget constraint under repayment. Revenues come from the distortionary labor tax and the inflation tax given money holdings m. The second constraint guarantees that the labor tax rate is non-negative.

The final constraint bounds the inverse of the inflation rate. Without any restriction, the monetary authority will always resort to the inflation tax and never tax labor income. Following Calvo (1978), we assume that money printing is bounded so that the effective inverse inflation rate cannot be lower than $\underline{\tilde{\pi}} > 0$. Chari, Christiano, and Eichenbaum (1998) impose a similar restriction.[22] Corsetti and Dedola (2013) imposes a convex cost of inflation to limit *ex post* money creation. Aguiar, Amador, Farhi, and Gopinath (2013) put the cost of inflation directly in agent's period utility.

If the government chooses to repay the debt, the real money tax base is given. Its policy is naturally biased toward inflation since taxing money holdings does not distort labor supply decisions of current money holders. If this tax revenue is sufficient to cover its obligations, there is no labor tax imposed. Else, if $\tilde{\pi} = \underline{\tilde{\pi}}$ does not generate enough revenue to cover its obligations, the government must impose a labor tax if it chooses to avoid default. This characterization is summarized in Lemma 3.

In the event of default, the choice of the inflation rate is welfare neutral given the specified social welfare function: when default occurs, monetary policy is implemented via lump-sum transfers which are purely redistributive. Thus we set $\tilde{\pi} = \underline{\tilde{\pi}}$ in the event of default. Setting inflation at this rate is consistent with the inflation chosen whenever the government is indifferent between default and repayment.

Lemma 3. *Under Assumption 2, given (A, m, i), the policy choices of the discretionary government are:*

- *if the government chooses to repay its debt, then*
  - $\tilde{\pi}^r = \max\left\{\underline{\tilde{\pi}}, \Pi(i,m)\right\}$, *where* $\Pi(i,m) = \frac{\nu^m m}{(1+i)b + \nu^m m}$,
  - $\tau > 0$ *and solves the government budget constraint if and only if* $\tilde{\pi}^r = \underline{\tilde{\pi}}$.
- *if the government chooses to default, then* $\tau = 0$ *and* $\tilde{\pi}^d = \underline{\tilde{\pi}}$.
- *the government chooses to default if and only if*

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} + \nu^m m(1 - \underline{\tilde{\pi}}) - (1+i)\underline{\tilde{\pi}}\theta b > 0$$

where τ *solves* (40) *given* $\tilde{\pi}^r = \underline{\tilde{\pi}}$.

Proof. If the government repays, its optimal choice of revenues between labor taxation and seignorage is driven by the different elasticity of these tax bases. As real money holding are predetermined, it will naturally first resort to the inflation tax and then complement with labor taxation if needed.[23] Hence, if the real money tax base is large enough to service debt, then its policy is $\tau = 0$ and $\tilde{\pi}^r = \frac{\nu^m m}{(1+i)b + \nu^m m}$ where this

[22] In the appendix of that paper, this restriction is rationalized by the presence of an alternative technology such that agents can bypass the cash-in-advance constraint. In effect, the return on this alternative technology pins down $\tilde{\pi}$. In our framework, the poor could store at a return of $r < 1$ instead of holding money. This alternative would put a floor on $\tilde{\pi}$.

[23] One could set a constrained optimization program and formally derive this result.

last expression is derived from the budget constraint. If resources from seignorage are not enough to service principal and interest on debt, then positive labor taxes are implemented: $\tau > 0$ if and only if $\tilde{\pi}^r = \underline{\tilde{\pi}}$.

Now consider the possibility of default, captured by $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$. Two cases need be distinguished. First, if the alternative under repayment does not require labor taxes to be raised, the default criterion $\Delta(\cdot)$ writes:

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} + \nu^m m(1 - \tilde{\pi}^r) - (1+i)\theta\tilde{\pi}^r b.$$

Under Assumption 2, this expression negative for all $\big(A, m, i, \theta\big)$, which rule out default when no labor taxes need to be raised[24]. Hence, the possibility to default arises when positive labor taxes $\tau > 0$ need to be raised to complement seignorage $\tilde{\pi}^r = \underline{\tilde{\pi}}$. Using these elements together with (16) and (17), one gets the expression for $\Delta(\cdot)$ stated above. ∎

4.3.2 Equilibrium Analysis

Building on this characterization, we investigate whether debt fragility remains under monetary discretion. First, we show that the multiplicity of interest rates consistent with the no-arbitrage condition (9) persists and interacts with inflation expectations.

Lemma 4. *Under Assumptions 2 and 3, under monetary discretion, there are multiple interest rates that solve the no-arbitrage condition (9).*

Proof. Using Assumption 3 and Lemma 3, there is a risk-free equilibrium with inflation expectations $\tilde{\pi}^e \geq \underline{\tilde{\pi}}$. This may arise with $\tau = 0$ and $\tilde{\pi}^e > \underline{\tilde{\pi}}$ or, from Lemma 3, with $\tau > 0$ and $\tilde{\pi}^e = \underline{\tilde{\pi}}$.

Suppose investors believe the government will default on its debt with positive probability. If the belief is self-fulfilling, then the optimal policy of the government must be to set the inflation level to $\underline{\tilde{\pi}}$ whether it reimburses its debt or default. Otherwise, resources from seignorage would be enough to cover principal and interests on debt for all realization of A, and default would be avoided. Hence, inflation expectations of agents are consistent with the best response of the government at $\tilde{\pi}^e = \underline{\tilde{\pi}}$. The no-arbitrage condition pricing public debt becomes:

$$(1+i)\underline{\tilde{\pi}}\Big(1 - F\big(\bar{A}(i)\big)\Big) = R \tag{42}$$

where $\bar{A}(i)$, defined in Lemma 1, is the boundary of the default region given i.

From Lemma 2, we know that there are at least two interest rates i that are consistent with this equilibrium condition, one of which carries a risk-premium and induces the government to default for some realization of A. Hence the initial pessimistic beliefs are self-fulfilling and support the existence of an interest rate that carries a positive probability of default. ∎

The key is that inflation expectations and probability of default are jointly linked by the anticipation of the best response of the discretionary government. In particular, the interest rate with a risk-premium that solves the no-arbitrage condition is systematically associated with the lowest real money tax base

[24] To see this, set $\theta = 0$, $m = 1$ and $\tilde{\pi}^r = 0$ and compare to the restriction in Assumption 2.

$m = \tilde{\pi}^e = \underline{\tilde{\pi}}$, which in turn prevents the central bank from inflating away the real value of debt. This lemma provides the basis for the existence of a sunspot equilibrium where the value of debt depends on beliefs of agents.

Proposition 4. *Under Assumptions 2 and 3, there is a sunspot equilibrium under discretion with the following properties:*

- *If $s_{-1} = s^o$, the interest rate to the government security is risk free and the treasury reimburses with probability 1, with either:*
  - *i. for all A, $\tilde{\pi}(A, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, \cdot) = 0$, $D(A, \cdot) = r$ and $\tilde{\pi}^e$ can take several values,*
  - *ii. for all A, $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, \cdot)$ solves the government budget constraint, $D(A, \cdot) = r$ and $\tilde{\pi}^e = \underline{\tilde{\pi}}$.*
- *If $s_{-1} = s^p$, the interest rate incorporates a risk-premium, for all A $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$, the treasury defaults on its debt for all $A < \bar{A}$ where $\bar{A} \in (A_l, A_h)$ and $\tilde{\pi}^e = \underline{\tilde{\pi}}$.*

Proof. We describe the optimal behavior of agents consistent with the equilibrium definition. This proof builds on Lemma 4 and the existence of several interest rates (and associated inflation expectations) consistent with the equilibrium definition.

If $s_{-1} = s^o$, then by Assumption 3, debt is risk free. Two cases need to be distinguished, depending on the real level of debt b. If $Rb < \frac{\nu^m}{4}$, then seignorage resources are enough to service the debt. Namely, there is $\tilde{\pi}^r \in (\underline{\tilde{\pi}}, 1)$ such that $Rb = \nu^m \tilde{\pi}^r(1 - \tilde{\pi}^r)$. Young agents form expectations of no default and $\tilde{\pi}^e = \tilde{\pi}^r$. They supply labor accordingly, young agents with low productivity save with money, young rich agents save via intermediated claims; the interest rate i on the government security satisfies the no-arbitrage condition (9) with $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^r$. The optimal policy of the government is then to set for all A $\tilde{\pi}(A, \cdot) = \tilde{\pi}^r$, $\tau(A, \cdot) = 0$ and repay the debt.

The multiplicity of inflation expectations $\tilde{\pi}^e$ and nominal interest rate i comes from the Laffer curve property of seignorage.[25] It reflects the interaction of inflation expectations and the optimal choice of inflation under discretion. If labor taxes are not imposed, then the government budget constraint simplifies to $Rb = \nu^m \tilde{\pi}^r(1 - \tilde{\pi}^r)$. This has multiple solutions for $\nu^m \underline{\tilde{\pi}}(1 - \underline{\tilde{\pi}}) < Rb < \frac{\nu^m}{4}$, as long as $\underline{\tilde{\pi}} < \frac{1}{2}$ (both sides of the Laffer curve are operative).

On the other hand, if $Rb \geq \frac{\nu^m}{4}$, then seignorage resources are not sufficient and taxes need be raised to service debt. Using Lemma 3 and Assumption 3, for all A, $\tilde{\pi}(A, \cdot) = \underline{\tilde{\pi}}$, $\tau(A, \cdot)$ solves the government budget constraint (12) and debt is repaid. Accordingly, young agents form expectations $P^d = 0$, $\tilde{\pi}^e = \underline{\tilde{\pi}}$, the government security is priced according to (9). In both cases, all markets clear.

For $s_{-1} = s^p$, we detail only the differences with the previous case. Independently of the level of b, young agents form rational expectations $P^d > 0$, and $\tilde{\pi}^e = \underline{\tilde{\pi}}$. The government security is priced accordingly. Given i and seignorage revenues $\nu^m \underline{\tilde{\pi}}(1 - \underline{\tilde{\pi}})$, there is a unique threshold $\bar{A}(i) > A_l$ such that the optimal policy is to raise labor taxes τ for all $A \geq \bar{A}(i)$ so as to satisfy the budget constraint and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F(\bar{A}(i))$. ∎

[25] Note that contrary to the fiscal Laffer curve, it is not possible for the central bank to "select" the efficient side of the Laffer curve, since real money holding is predetermined to the decisions of the government.

Does monetary discretion provide a shield against debt fragility? Can the government inflate the real value of debt and generate additional resources to service its debt? The answer is negative. Indeed, when pessimism hits the economy, the interplay between inflation expectations and real money tax base *corners* the central bank into a high inflation regime with no more capacity to inflate debt or provide additional resources to the treasury. Hence, under monetary discretion, the sunspot shock to investors confidence triggers a joint shift in inflation expectations and debt sustainability.

Note that embedded in Proposition 4 is the potential for multiplicity in the inflation tax in the discretion regime. In the event debt repayment is preferred, there may exist multiple rates of money creation and hence inflation that generates the necessary revenue. In this sense, the multiplicity of debt valuations is supplemented by strategic uncertainty over the rate of inflation even when investors are optimistic about debt repayment.

The equilibria characterized in Proposition 4 depend on $\underline{\tilde{\pi}}$. If the ceiling on maximal inflation is lowered (i.e. $\underline{\tilde{\pi}}$ is higher), then the revenue collected from seignorage will change. Whether revenue increases or decreases depends on the response of money demand, i.e. on which side of the Laffer curve is the economy operating. For this model, the upper slope of the Laffer curve corresponds to $\underline{\tilde{\pi}} > \tilde{\pi}^L$.

Proposition 5. *In the sunspot equilibrium characterized in Proposition 4, if $\underline{\tilde{\pi}} > \tilde{\pi}^L$, then a reduction in the inflation ceiling (i.e. an increase in $\underline{\tilde{\pi}}$) will (i) increase the magnitude of taxes when $s_{-1} = s^o$ and (ii) increase the probability of default under $s_{-1} = s^p$, if and only if the equilibrium is locally stable.*

Proof. An increase in $\underline{\tilde{\pi}}$ will, if $\underline{\tilde{\pi}} > \frac{1}{2}$, lower seignorage revenues. From Proposition 4, if $s_{-1} = s^o$ and $Rb \geq \frac{\nu^m}{4}$, the government sets $\tilde{\pi}(\cdot) = \underline{\tilde{\pi}}$ and relies on labor taxes to pay the remainder of its obligations. If $\underline{\tilde{\pi}}$ increases, then the government have to increase τ to finance its debt obligation.

In the case $s_{-1} = s^p$, the inflation rate is $\tilde{\pi}(\cdot) = \underline{\tilde{\pi}}$. Using the result from Proposition 3, we get that an increase in $\underline{\tilde{\pi}}$, if $\underline{\tilde{\pi}} > \tilde{\pi}^L$, i.e. a reduction in the inflation ceiling on the upward slopping part of the Laffer curve, will increase the default threshold if and only if the equilibrium is locally stable (under best response dynamics). ∎

5 Stabilization through Commitment: Leaning Against the Winds

These results make clear that the monetary authority may be unable to prevent debt fragility. If there is a commitment to an unconditional inflation tax (or inflation target), the fiscal authority will still need to raise revenue, thus exposing the debt to multiple valuations. If the monetary authority has complete discretion, then it will use the inflation tax to raise revenue *ex post* and again fiscal tools may be needed to finance debt repayments. Consequently, in periods of low productivity, the tax burden can become excessive leading to default. In equilibrium, the valuation of debt will be subject to investor sentiments in a sunspot equilibrium.

This section returns to the commitment case. Instead of imposing an inflation target, we allow the central bank to choose a state-contingent inflation policy that alters the real debt burden and distributes resources from seignorage across states. By carefully exploiting the distribution of realized inflation, the central bank can provide a shield against debt fragility.

Suppose the monetary authority commits to a rule given by $\tilde{\pi}(A, i, s_{-1})$: the rate of inflation in the current period depends on current productivity, the interest rate on outstanding debt as well as the sunspot realization from previous period.[26] This rule is devised with a couple of key properties. First, to induce agents to hold money, the rule will deliver a target rate of inflation. Second, it will support the fundamental equilibrium by using monetary tools to counter pessimistic expectations so that equilibria with strategic uncertainty no longer exist.[27] In this way, the monetary authority responds to variations in current beliefs, reflected in the sunspot and the interest rate, by appropriately setting policy for the future. Importantly, the policy intervention responds to variations in productivity: the rate of inflation is inversely related to current productivity if investors were pessimistic in the previous period. Specifically, when A is high, the rate of inflation is relatively low and fiscal policy, through the setting of tax rates, bears more of the burden of financing debt obligations. But during times of low productivity, when default is likely, the monetary authority inflates the real value of debt and generates seignorage revenue. Both effects allow the fiscal authority to set low taxes and avoid default.

We first describe the desired properties of this policy, derive its existence and properties in Lemma 5. Then, we characterize the equilibrium of the economy under $\tilde{\pi}(A, i, s_{-1})$ and argue that such monetary policy rule stabilize debt valuations.

Specifically, suppose the central bank commits to a rule in which $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$ for all (A, i): under optimism, there is an inflation target as in *monetary delegation*. The target of $\tilde{\pi}^*$ is independent of both current productivity A and the interest rate on debt. When $s_{-1} = s^p$, the central bank implements a state dependent (on (A, i)) monetary policy.[28]

First, given pessimism the policy rule anchors inflation expectations: $\tilde{\pi}(A, i, s^p)$ meets the inflation target *on average*:

$$\int_A \tilde{\pi}(A, i, s^p) dF(A) = \tilde{\pi}^*, \tag{43}$$

for all i. Combined with the policy under optimism, $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$, unconditional inflation expectations are anchored at $\tilde{\pi}^*$.

Second, $\tilde{\pi}(A, i, s^p)$ deters partial default: given A and i, the treasury either reimburses its debt with probability 0 or 1. For lower values of debt obligations, the fiscal authority will choose to repay its debt, for all A. For high values of these obligations, the fiscal authority will default, again for all A. Of course, the size of the debt obligations are determined in equilibrium, based upon investor beliefs and central bank policy. Formally, Lemma 5 establishes that there is a monetary policy rule under pessimism that satisfies these two properties.

[26] This commitment is independent of other elements of the state vector.

[27] To be clear, the policy is designed to eliminate interior equilibria. The equilibrium with certain default remains.

[28] In the equilibrium constructed below, optimism is equivalent to an interest rate satisfying $(1 + i)\tilde{\pi}^* = R$. Hence there is only one interest rate conditional on optimism. If there is pessimism, we specify monetary policy as a function of the interest rate on the outstanding debt. An alternative way to characterize the sunspot equilibrium, particularly what happens off the equilibrium path, would be to write the equilibrium conditions solely as a function of the interest rate. This is used in the discussion of policy implementation.

Lemma 5. *Given an inflation target $0 < \tilde{\pi}^* \leq 1$, there is a monetary policy rule $\tilde{\pi}(A, i, s^p)$ that satisfies the inflation target and deters partial default. Moreover, $\tilde{\pi}(A, i, s^p) > 0$ for all (A, i) and is increasing in A.*

Proof. We derive a state-contingent monetary policy rule $\tilde{\pi}(A, i, s^p)$ that satisfies (43) and deters partial default. Consider the case $\theta = 0$, where all debt is held abroad, and $\nu^m \approx 0$, which makes seignorage a negligible source of income for the fiscal authority. This simplified framework outlines clearly that the capacity of the central bank to influence the default decision of the treasury does not primarily rely on providing more or less resources, but rather on its capacity to alter the real return to debt across states. The proof is extended to the general case $\theta \geq 0$ and $\nu^m \geq 0$ in Appendix 7.2.

Given $s_1 = s^p$, we focus on the dependence of inflation on the interest rate on outstanding debt. There is a state contingent rule $\tilde{\pi}(A, i, s^p)$, denoted $\tilde{\pi}^p_A$ in the following analysis.[29] This rule induces a unique interest rate cut-off i^δ such that if $i < i^\delta$ then the fiscal authority is induced to repay its debt for all A, i.e. with probability 1. If $i > i^\delta$, then the fiscal authority defaults for all A, i.e. with probability 1. For $i = i^\delta$, the fiscal authority is indifferent between repayment and default for all A. This condition for indifference is:

$$\Delta(A, i^\delta, m, \tau, \tilde{\pi}^p_A) = W^d(\cdot) - W^r(\cdot) = 0 \qquad \forall A, \tag{44}$$

where $m = \tilde{\pi}^*$ using the inflation target condition (43), $\tilde{\pi}^p_A$ is defined below and τ satisfies the government budget constraint (14) given $(A, i^\delta, \tilde{\pi}^p_A)$:

$$(1 + i^\delta)\tilde{\pi}^p_A b = A^2(1 - \tau)\tau. \tag{45}$$

Using $\theta = 0$ and $\nu^m \approx 0$, (44) implies $\tau = \gamma$ for all A. From the government budget constraint:

$$\tilde{\pi}^p_A = \frac{A^2(1 - \gamma)\gamma}{(1 + i^\delta)b} \qquad \forall A. \tag{46}$$

Applying the inflation target requirement (43), the nominal interest rate cut-off i^δ is:

$$1 + i^\delta = \frac{(1 - \gamma)\gamma}{\pi^* b} \int_A A^2 dF(A), \tag{47}$$

which gives:

$$\tilde{\pi}^p_A = \frac{A^2 \pi^*}{\int_A A^2 dF(A)}. \tag{48}$$

To verify that this monetary rule deters partial default, note:

$$\frac{d\Delta(A, i, m, \tau, \tilde{\pi}^p_A)}{di} = A^2(1 - \tau)\frac{d\tau}{di}, \tag{49}$$

where $\frac{d\tau}{di} = \frac{\tilde{\pi}^p_A b}{A^2(1-2\tau)} > 0$ from (45). As $\Delta(A, i^\delta, \tilde{\pi}^*, \tau, \tilde{\pi}^p_A) = 0$ for all A, we get that for all A and all $i < i^\delta$, $\Delta(\cdot) < 0$ and for all $i > i^\delta$, $\Delta(\cdot) > 0$. Hence there is no nominal interest rate $i > 0$ that induces the fiscal authority to partially default on its debt. Finally, from (48), we get $\tilde{\pi}^p_A > 0$ and $\frac{d\tilde{\pi}^p_A}{dA} > 0$. ∎

The lemma establishes two critical properties of $\tilde{\pi}(A, i, s^p)$. First, for all A, $\tilde{\pi}(A, i, s^p) > 0$, which rules out any issue of demonetization of the economy and state-contingent complete default via inflation. Second,

[29] So, given pessimism, the intervention depends on A but not directly on i.

the policy rule is countercyclical: $\tilde{\pi}(A, i, s^p)$ is increasing in A, i.e. the lower the technology realization, the higher inflation. This policy distributes resources from seignorage across states, with high seignorage revenue $\nu^m \tilde{\pi}^*(1 - \tilde{\pi}^p_A)$ for low realizations of A. Hence, even if seignorage revenues are not essential to rule out equilibria with partial default, the policy further contributes to lower the fiscal burden in states where fiscal needs are the highest, i.e. the induced fiscal policy is also countercyclical.[30]

When the central bank commits to $\tilde{\pi}(A, i, s_{-1})$, there is a unique price for debt, namely the fundamental price under inflation targeting. That is, there are no sunspot equilibria. Formally,

Proposition 6. *Under Assumptions 2, 3, when the monetary authority commits to $\tilde{\pi}(A, i, s_{-1})$, with $\tilde{\pi}(A, i, s^p)$ given in Lemma 5, debt is uniquely valued and risk-free. Debt fragility is eliminated.*

Proof. Under Assumption 3, there is a risk-free outcome under strict inflation target $0 < \tilde{\pi}^* \leq 1$. Hence, there is an equilibrium nominal interest rate $\underline{i}$ under optimism that satisfies $(1 + \underline{i})\tilde{\pi}^* = R$. Now suppose that the monetary authority commits to $\{\tilde{\pi}^p_A\}$ under pessimism as defined in Lemma 5. We verify that the best response of the treasury is to repay its debt for all A and that the equilibrium interest rate is $\underline{i}$.

A central property of $\{\tilde{\pi}^p_A\}$ is that it delivers the inflation target on average. By continuity and monotonicity of $\tilde{\pi}^p_A$ in A, there is a realization $\tilde{A}$ such that $\tilde{\pi}^p_{\tilde{A}} = \tilde{\pi}^*$. In this case, the best-response of the fiscal authority is to raise taxes and repay its debt. Second, $\{\tilde{\pi}^p_A\}$ is such that if the fiscal authority repays its debt with positive probability, it repays its debt with probability 1. Hence, under $\{\tilde{\pi}^p_A\}$, the fiscal authority repays its debt for all A and debt is risk-free. Finally, the no-arbitrage condition under inflation target $\tilde{\pi}^*$ uniquely pins down the nominal interest rate. Hence, under $\{\tilde{\pi}^p_A\}$, the nominal interest rate is $\underline{i}$:

$$(1 + \underline{i}) \int_A \tilde{\pi}^p_A dF(A) = (1 + \underline{i})\tilde{\pi}^* = R \tag{50}$$

∎

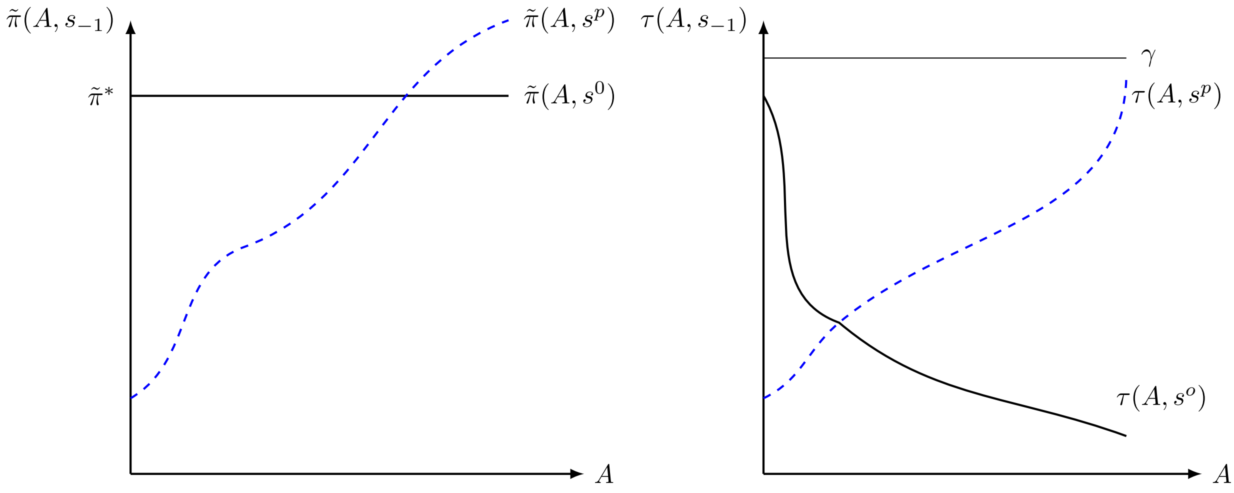
This proposition makes clear that the commitment of the central bank rules out the effect of pessimism on the value of debt. The key to this result is the relaxation of the incentive to default by the fiscal authority through the provision of seignorage revenue and the erosion of the real return to debt in low productivity states.

Figure 2 displays the monetary policy rule and the induced tax policy, as described in Proposition 6. In the case $s_{-1} = s^p$, note the distribution of inflation over realization of A: for low A, high inflation, i.e. low real value of debt and high seignorage revenue. Hence, in case of pessimism, the monetary authority implements a countercylical policy that stabilizes the price of debt and provides fiscal relief for low values of A, compensated by lower inflation for higher realizations of A. A critical element of this policy is the commitment of the central bank so that inflation expectations are anchored and the real money tax base is not sensitive to variation in private agents sentiment. It illustrates how the central bank can alter the real value of debt, and incidentally distribute income from seignorage, so as to contain the fiscal pressure that weights on the fiscal authority.

As written, the monetary intervention depends jointly on the sunspot from the previous period as well as the interest on outstanding debt. Along the equilibrium path, from Proposition 6, only the fundamental

[30]In fact, the proof focuses on the case of ν^m near zero.

Figure 2: State Dependent Monetary and Fiscal Policy



(a) Monetary Policy

(b) Fiscal Policy

The left panel represents the state dependent monetary policy to which the central bank commits. The right panel represents the induced fiscal policy. The dependence of the policies on the sunspot and realized productivity are displayed.

price of debt will be observed. Though extraneous uncertainty may still exist, it will not be reflected in the equilibrium interest rates. With this in mind, it may be more natural to condition monetary interventions on interest rates so that along the equilibrium path, no actual interventions are needed. But, the monetary authority stands ready to intervene in response to higher interest rates that reflect investor pessimism. This is, in effect, a threat of the monetary authority off the equilibrium path to intervene either to support the fiscal authority or, if interest rates are too high, to allow default with probability one. In this sense, the monetary authority leans against the winds of pessimism as well as those associated with low productivity.

6 Conclusions

The goal of this paper was to determine whether monetary policy enhances or mitigates fiscal fragility. Cast as a real economy, the basic environment has fragile debt: there are multiple valuations of government debt depending on the beliefs of investors.

The effects of introducing monetary interventions depends on the commitment of the central bank. If the central bank is committed to an inflation target, then fiscal fragility remains. If the central bank is allowed full discretion, then the presence of an inelastic source of finance through seignorage is internalized by private agents. Any temptation to inflate the real value of debt is anticipated. Henceforth, fiscal fragility remains, and goes together with a shift in inflation expectations.

Finally, we analyze how a committed central bank can deter run on government security, by designing a specific monetary policy rule. We devise a state contingent intervention that eliminates pessimistic evaluations of government debt. The policy requires the monetary authority to implement a countercyclical policy, that erodes the real value of debt and provides resources, through seignorage, in times of low productivity and thus low revenue. By supporting the fiscal authorities in these states, the incentive for default is eliminated.

A number of extensions are worth consideration. First, the paper studies the extremes of commitment and discretion. An interesting middle case would be stochastic commitment. A government acting in period t would be allowed to adjust its policy in period $t+1$ with a probability less than one. This partial commitment would create a cost of high inflation and thus enrich the analysis.

Second, the model is dynamic but the fiscal policy is within a generation. Thus we have assumed away the possibilities of debt turnover and intertemporal punishments for default.

Third, our analysis has underlined that the capacity of the central bank to stabilize debt valuations rely the issuance of non contingent nominal assets labelled in domestic currency. It does not apply to real, indexed debt or debt issued in foreign currency. Allowing governments this choice would be of interest.

Finally, as in many other studies, the outcome with discretion imposes an upper bound on inflation. Providing further micro foundations for this bound remains an open area. Perhaps a political economy model that stresses the redistribution aspects of labor income vs inflation tax would be a productive approach.

7 Appendix

7.1 Welfare under Repayment and under Default

As explained in section (2.2.1), the repayment vs. default decision in this environment is a discrete choice that affects only the welfare of old agents. Hence, the welfare criteria of interest for $D \in \{r, d\}$ is:

$$W^D(A, i, m, \tau, \tilde{\pi}) = \nu^m \Big[c_o^m(h) - \frac{n_o^m(h)^2}{2}\Big] + \nu^I \Big[c_o^I(h) - \frac{n_o^I(h)^2}{2}\Big].$$

Using the optimal labor supply decisions derived in (4) and (10), we get the following consumption and labor supply vectors:

$$\begin{aligned} c_o^m(r) &= An_o^m(r)(1-\tau) + m\tilde{\pi}^r & \qquad c_o^m(d) &= An_o^m(d)(1-\gamma) + m\tilde{\pi}^d + t \\ n_o^m(r) &= A(1-\tau) & n_o^m(d) &= A(1-\gamma) \\ c_o^I(r) &= An_o^I(r)(1-\tau) + (1+i)\tilde{\pi}^r b^I + Rk & c_o^I(d) &= An_o^I(d)(1-\gamma) + Rk + t \\ n_o^I(r) &= A(1-\tau) & n_o^I(d) &= A(1-\gamma). \end{aligned}$$

Using $\nu^I b^I = \theta b$, one can solve for k, the risk-free component of individual portfolio of rich agents from their budget constraint:

$$\begin{aligned} zn_y^I &= Rz^2 = b^I + k + \Gamma \\ \Rightarrow \nu^I Rk &= \nu^I R\big(Rz^2 - \Gamma\big) - R\theta b. \end{aligned}$$

We derive the expressions for $W^r(\cdot)$ and $W^d(\cdot)$:

$$W^r(A,i,m,\tau,\tilde{\pi}^r) = \frac{\big[A(1-\tau)\big]^2}{2} + \nu^m m\tilde{\pi}^r + \big((1+i)\tilde{\pi}^r - R\big)\theta b + \nu^I R(Rz^2-\Gamma)$$
$$W^d(A,i,m,\tilde{\pi}^d) = \frac{\big[A(1-\gamma)\big]^2}{2} + \nu^m m\tilde{\pi}^d - R\theta b + \nu^I R(Rz^2-\Gamma) + T$$

where τ solves the government budget constraint under repayment and $T = \nu^m m(1-\tilde{\pi}^d)$ is a lump sum transfer that implements monetary policy $\tilde{\pi}^d$ under default.

Default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

7.2 An Effective Monetary Policy Rule

This section details the proof of Lemma 5 in the general case $\theta \in [0,1]$ and $\nu^m \geq 0$.

We adopt the following notations. Given $m = \tilde{\pi}^e = \tilde{\pi}^*$, where $\tilde{\pi}^* = \int_A \tilde{\pi}_A dF(A)$ is the inflation target of the central bank, the expression that captures the discretionary default decision of the treasury writes:

$$\begin{aligned}\Delta(A,i,\tilde{\pi}^*,\tau,\tilde{\pi}_A) &= W^d(\cdot) - W^r(\cdot)\\ &= \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} + \nu^m\tilde{\pi}^*(1-\tilde{\pi}_A) - (1+i)\tilde{\pi}_A\theta b,\end{aligned}$$

where τ solves the government budget constraint given $\tilde{\pi}_A$:

$$G(A,i,\tilde{\pi}^*,\tau,\tilde{\pi}_A) = A^2(1-\tau)\tau + \nu^m\tilde{\pi}^*(1-\tilde{\pi}_A) - (1+i)\tilde{\pi}_A b = 0.$$

Moreover, in the economy with $\theta > 0$, default occurs for two reasons: either it is the best response of the treasury: $\Delta(\cdot) > 0$, or the fiscal capacity of the country cannot service debt, since $\tau \leq \frac{1}{2}$.

We want to show that there is a unique state-dependent inflation policy $\{\tilde{\pi}^p_A\}$ and an induced interest rate cut-off i^δ such that the policy delivers the inflation target on average, and, if the central bank commits to $\{\tilde{\pi}^p_A\}$, then the fiscal authority services its obligation for all A if and only if $i < i^\delta$.

We proceed in two steps: first we show that for any i^t, there is a unique policy $\{\tilde{\pi}_A(i^t)\}$ such that the treasury reimburses its debt if and only if $i < i^t$. Second, we show that there is a unique i^δ such that $\{\tilde{\pi}_A(i^\delta)\}$ satisfies the inflation target. The desired policy is given by $\tilde{\pi}^p_A = \tilde{\pi}_A(i^\delta)$ for all A.

Part I. Consider a nominal interest rate i^t such that $1 + i^t > 0$ and $A \in [A_l, A_u]$.

(i) The following elements establish that there is a unique inflation level $\tilde{\pi}_A(i^t)$ such that the fiscal authority is indifferent between repayment and default.

First, there is an inverse inflation rate $\tilde{\pi}^1_A(i^t)$ such that debt is serviced with no taxes on labor income.

$$G(A,i^t,\pi^*,\tau,\tilde{\pi}^1_A(i^t)) = 0 \Rightarrow \tau = 0.$$

In this case, using Assumption 2[31], $\Delta(\cdot) < 0$. Formally, solving the government budget constraint with $\tau = 0$:

$$\tilde{\pi}^1_A(i^t) = \frac{\nu^m\tilde{\pi}^*}{\nu^m\tilde{\pi}^* + (1+i^t)b} > 0.$$

[31] This Assumption ensures that default is not desired when no tax need be raised. For a formal argument, see the proof of Lemma 3.

Similarly, the central bank can set the inverse inflation rate to $\tilde{\pi}_A^2(i^t)$ so that if the treasury desires to service its debt, it has to set $\tau = \frac{1}{2}$. Formally:

$$\tilde{\pi}_A^2(i^t) = \frac{\frac{A^2}{4} + \nu^m \tilde{\pi}^*}{\nu^m \tilde{\pi}^* + (1+i^t)b}.$$

Importantly, for any inflation rate between these two cases, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the tax rate to service debt. Differentiating the government budget constraint w.r.t. τ and $\tilde{\pi}_A$, one gets:

$$\forall \tilde{\pi}_A \in [\tilde{\pi}_A^1(i^t), \tilde{\pi}_A^2(i^t)],\ \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m \tilde{\pi}^* + (1+i^t)b}{A^2(1-2\tau)} > 0.$$

Moreover, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the value of $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$:

$$\frac{d\Delta(\cdot)}{d\tilde{\pi}_A} = \frac{1-\tau}{1-2\tau}(\nu^m \tilde{\pi}^* + (1+i^t)b) - (\nu^m \tilde{\pi}^* + (1+i^t)\theta b) > 0,$$

since $\frac{1-\tau}{1-2\tau} > 1$ for $\tau \in [0, \frac{1}{2})$.

Hence, there is a unique $\tilde{\pi}_A(i^t)$ that has the desired property to make the treasury indifferent between repayment and default. Especially,

- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)\big) > 0$, then $\tilde{\pi}_A^1(i^t) < \tilde{\pi}_A(i^t) < \tilde{\pi}_A^2(i^t)$,
- if $\Delta\big(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)\big) \leq 0$, then $\tilde{\pi}_A(i^t) = \tilde{\pi}_A^2(i^t)$.

(ii) Next, we verify that for any $i < i^t$, the fiscal authority services its debt, otherwise for any $i > i^t$, it defaults. Given $\tilde{\pi}_A(i^t)$, we have:

$$\frac{d\Delta(\cdot)}{di} = A^2(1-\tau)\frac{d\tau}{di} - \tilde{\pi}_A(i^t)\theta b = \frac{1-\tau}{1-2\tau}\tilde{\pi}_A(i^t)b - \tilde{\pi}_A(i^t)\theta b > 0.$$

(iii) Also, we establish the following properties of $\tilde{\pi}_A(i^t)$:

$$\frac{d\tilde{\pi}_A(i^t)}{dA} > 0 \qquad \frac{d\tilde{\pi}_A^p(i^t)}{di^t} < 0.$$

If $\tilde{\pi}_A(i^t) = \tilde{\pi}_A^2(i^t)$, these properties are straightforward. In the case $\tilde{\pi}_A(i^t) < \tilde{\pi}_A^2(i^t)$, first differentiate the government budget constraint w.r.t. $(A, i, \tau, \tilde{\pi}_A)$ to get:

$$\frac{d\tau}{dA} = -\frac{2(1-\tau)\tau}{A(1-2\tau)} \qquad \frac{d\tau}{di} = \frac{\tilde{\pi}_A b}{A^2(1-2\tau)} \qquad \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m \tilde{\pi}^* + (1+i)b}{A^2(1-2\tau)}$$

Then differentiate $\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A)$ w.r.t to its arguments and using the derivative of τ w.r.t $(A, i, \tilde{\pi}_A)$, one gets:

$$\Big[A(1-\gamma)^2 - A\frac{(1-\tau)^2}{1-2\tau}\Big]dA + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^m \tilde{\pi}^* + (1+i)b\big) - \big(\nu^m \tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0$$
$$\Big[\frac{1-\tau}{1-2\tau}\tilde{\pi}_A b - \tilde{\pi}_A \theta b\Big]di + \Big[\frac{1-\tau}{1-2\tau}\big(\nu^m \tilde{\pi}^* + (1+i)b\big) - \big(\nu^m \tilde{\pi}^* + (1+i)\theta b\big)\Big]d\tilde{\pi}_A = 0$$

Since $\frac{1-\tau}{1-2\tau} > \frac{(1-\tau)^2}{1-2\tau} > 1$ for all $0 \leq \tau \leq \frac{1}{2}$ and $0 \leq \theta \leq 1$, we get the desired result.

(iv) Finally, the limits behavior of $\tilde{\pi}_A(i^t)$ are derived from the inequality

$$\tilde{\pi}_A^1(i^t) < \tilde{\pi}_A(i^t) \leq \tilde{\pi}_A^2(i^t),$$

which gives $\lim_{i^t \to +\infty} \tilde{\pi}_A(i^t) = 0$ and $\lim_{i^t \to -1} \tilde{\pi}_A(i^t) > 1$.

Part II. By applying the inflation target requirement (43), we show that there is a unique $i^\delta > 0$ such that:

$$\int_A \tilde{\pi}_A(i^\delta) dF(A) = \tilde{\pi}^*.$$

Note $H(i) = \int_A \tilde{\pi}_A(i) dF(A)$, which is defined for all i such that $1 + i > 0$. The properties of $\tilde{\pi}_A(i)$ naturally convey to $H(i)$: $H(i)$ is strictly decreasing in i; $\lim_{i \to +\infty} H(i) = 0$; $\lim_{i \to -1} H(i) > 1$.

Hence there is a unique i^δ such that $H(i^\delta) = \tilde{\pi}^*$.

Overall, the monetary policy rule $\{\tilde{\pi}_A^p\}$ that meets the inflation target and systematically deters partial default, exists, is unique, and satisfies:

$$\forall\ A\ \tilde{\pi}_A^p = \tilde{\pi}_A^p(i^\delta).$$

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